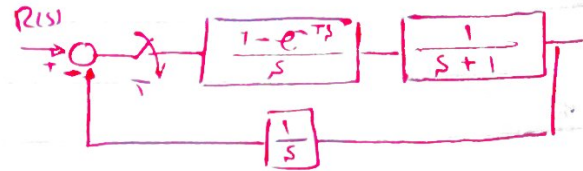


Sheet 3

1] For the given System

Required : output CLK for unit step input
Discuss System Stability



$$G(s) = \frac{1 - e^{-Ts}}{s(s+1)}, \quad G_H(s) = \frac{1 - e^{-Ts}}{s^2(s+1)} \quad \text{sol}^n$$

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G_H(z)}$$

$$\begin{aligned} \rightarrow G(z) &= \mathcal{Z} \left[\frac{1 - e^{-Ts}}{s(s+1)} \right]_{T=1} = (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s} - \frac{1}{s+1} \right] = (1 - z^{-1}) \mathcal{Z} [1 - e^{-t}] \\ &= (1 - z^{-1}) \left[\frac{z}{z-1} - \frac{z}{z - e^{-1}} \right] = 1 - \frac{z-1}{z-0.368} = \boxed{\frac{0.632}{z-0.368}} \end{aligned}$$

$$\begin{aligned} \rightarrow G_H(z) &= \mathcal{Z} \left[\frac{1 - e^{-Ts}}{s^2(s+1)} \right] = (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s^2(s+1)} \right] = (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right] \\ &= (1 - z^{-1}) \mathcal{Z} [t - 1 + e^{-t}] = \frac{z-1}{z} \left[\frac{z}{(z-1)^2} - \frac{z}{(z-1)} + \frac{z}{z - e^{-1}} \right] \\ &= \frac{1}{z-1} - 1 + \frac{z-1}{z-0.368} = \frac{(z-0.368)(z-1)(z-0.368) + (z-1)^2}{(z-1)(z-0.368)} \\ &= \frac{z - 0.368 - z^2 + 1.368z - 0.368 + z^2 - 2z + 1}{(z-1)(z-0.368)} = \boxed{\frac{0.368z + 0.264}{(z-1)(z-0.368)}} \end{aligned}$$

$$\therefore \frac{C(z)}{R(z)} = \frac{\frac{0.632}{z-0.368}}{1 + \frac{0.368z + 0.264}{(z-1)(z-0.368)}} = \frac{0.632(z-1)}{(z-1)(z-0.368) + (0.368z + 0.264)}$$

$$\frac{C(z)}{R(z)} = \boxed{\frac{0.632(z-1)}{z^2 - z + 0.632}}$$

For unit step input $R(z) = \frac{z}{z-1}$

$$\therefore C(z) = \frac{0.632(z-1)}{z^2 - z + 0.632} \cdot \frac{z}{z-1} = \frac{0.632 z}{z^2 - z + 0.632}$$

Remember

$$Z[a^k f(k)] = F(z) \Big|_{z=\frac{z}{a}} \longrightarrow Z[a^k \sin \omega t] = \frac{\frac{z}{a} \sin \omega T}{\left(\frac{z}{a}\right)^2 - 2\left(\frac{z}{a}\right) \cos \omega T + 1}$$

for $T=1$

$$= \frac{z a \sin \omega T}{z^2 - 2 z a \cos \omega T + a^2}$$

$$\therefore C(z) = \frac{0.632 z}{z^2 - z + 0.632} \quad \text{By analogy}$$

$$\therefore a^2 = 0.632 \longrightarrow a = 0.795$$

$$2a \cos \omega = 1 \longrightarrow \cos \omega = \frac{1}{2 \times 0.795} = 0.629 \longrightarrow \omega = 51.03^\circ = 0.891 \text{ rad/s}$$

$$\therefore a \sin \omega = 0.795 \times 0.777 = 0.618$$

$$\therefore C(z) = \frac{0.632}{0.618} \cdot \frac{(0.618) z}{z^2 - z + 0.632} \quad \text{to match the form}$$

$\xrightarrow{1.022}$ $\xrightarrow{a \sin \omega}$ $\xrightarrow{2a \cos \omega}$ $\xrightarrow{a^2}$

$$\therefore C(k) = 1.022 (0.795)^k \sin(0.891 k) \quad \#$$

→ System Stability

$$\frac{C(z)}{R(z)} = \frac{0.632(z-1)}{z^2 - z + 0.632}$$

$$\text{c/c eqn } z^2 - z + 0.632 = 0 \longrightarrow z_{1,2} = \frac{1 \pm \sqrt{1 - 4 \times 0.632}}{2} = 0.5 \pm j0.618$$

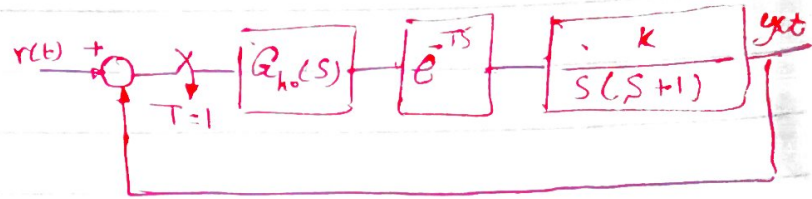
$$\|z_{1,2}\| = \sqrt{0.5^2 + 0.618^2} = 0.632 < 1$$

the poles located inside the unit circle → Stable System

For the given System

- Determine the O.L.T.F
- What is the type of the System
- Calculate e_{ss} for unit ramp
- Calculate $y(\infty)$ for unit step

Assume $k=0.1$



solⁿ

$$\begin{aligned}
 4) \text{ The O.L.T.F } G(z) &= \mathcal{Z} \left[\frac{1-e^{-Ts}}{s} \cdot e^{-Ts} \cdot \frac{k}{s(s+1)} \right] \\
 &= k(1-z^{-1}) z^{-1} \mathcal{Z} \left[\frac{1}{s^2(s+1)} \right] \\
 &= k \frac{(z-1)}{z^2} \mathcal{Z} \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right] \\
 &= k \frac{(z-1)}{z^2} \left[\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}} \right]_{T=1} \\
 &= \frac{k}{z} \left[\frac{1}{z-1} - 1 + \frac{z-1}{z-0.368} \right] \\
 &= \frac{k}{z} \left[\frac{(z-0.368) - (z-1)(z-0.368) + (z-1)^2}{(z-1)(z-0.368)} \right] \\
 &= \frac{k}{z} \left[\frac{z-0.368 - z^2 + 1.368z - 0.368 + z^2 - 2z + 1}{(z-1)(z-0.368)} \right] \\
 &= \frac{k(0.368z + 0.264)}{z(z-1)(z-0.368)}
 \end{aligned}$$

ii) The System is type 1

note: type of discrete system is the order of $(z-1)$ term

(iv) For unit ramp input $e_{ss} = \frac{1}{k_v}$

$$k_v = \frac{1}{T} \lim_{z \rightarrow 1} (1-z^{-1}) G(z)$$

$$= \lim_{z \rightarrow 1} \frac{z-1}{z} \cdot \frac{k(0.368z+0.264)}{z(z-1)(z-0.368)} = \frac{k(0.632)}{1-0.368} = k$$

$$e_{ss} = \frac{1}{k} \quad \text{for } k=0.1 \rightarrow \boxed{e_{ss} = 10}$$

(v) $y(\infty) = \lim_{t \rightarrow \infty} y(t) = \lim_{z \rightarrow 1} \frac{(z-1)}{z} Y(z)$

$$\frac{Y(z)}{R(z)} = \frac{G(z)}{1+G(z)} = \frac{k(0.368z+0.264)}{z(z-1)(z-0.368) + k(0.368z+0.264)}$$

$$= \frac{k(0.368z+0.264)}{z^3 - 1.368z^2 + 0.368(1+k)z + 0.264k} \quad \text{C.L.T.F}$$

For unit step input $\rightarrow R(z) = \frac{z}{z-1}$

$$Y(z) = \frac{k(0.368z+0.264)}{z^3 - 1.368z^2 + 0.368(1+k)z + 0.264k} \cdot \frac{z}{z-1} \quad (\text{Type 1})$$

$$y_{ss} = y(\infty) = \lim_{z \rightarrow 1} \frac{(z-1) \cdot k(0.368z+0.264) \cdot z}{z(z^3 - 1.368z^2 + 0.368(1+k)z + 0.264k)(z-1)}$$

$$= \frac{0.632 \times k}{1 - 1.368 + 0.368 + 0.368k + 0.264k} = \frac{0.632k}{0.632k} = 1$$

Another solution:

- For unit step input $e_{ss} = \frac{1}{1+k_p}$, $k_p = \lim_{z \rightarrow 1} G(z) = \infty$

$\rightarrow e_{ss} = \text{Zero}$

$\therefore e_{ss} = r_{ss} - y_{ss}$

$y_{ss} = r_{ss} - e_{ss} = 1 - 0 = 1$